

Abstract

A multiple-scales approach is used to approximate the effects of nonparallelism and streamwise surface curvature on the growth of stationary crossflow vortices in incompressible, three-dimensional boundary layers. The results agree with the results predicted by solving the parabolized stability equations in regions where the nonparallelism is sufficiently weak. As the nonparallelism increases, the agreement between the two approaches worsens. An attempt has been made to quantify the nonparallelism on flow stability in terms of a nondimensional number that describes the rate of change of the mean flow relative to the disturbance wavelength. We find that the above nondimensional number provides useful information about the adequacy of the multiple-scales approximation for different disturbances for a given flow geometry, but the number does not collapse data for different flow geometries onto a single curve.

1 Introduction

In flows of aeronautical interest, laminar-turbulent transition often occurs after small disturbances in the external flow are internalized into the boundary layer in the form of instability waves. These instability waves grow; eventually nonlinear effects become important; secondary and higher instabilities occur; and the spectrum of disturbances rapidly spreads as the flow approaches a turbulent state. The longest extent of this process is typically dominated by the linear growth of the instability waves. Hence, the correct calculation of the growth of the unstable waves is critical to any rational approach to predict the onset of transition.

The evolution of an instability wave is often computed as the integral of its growth rate as it propagates along the airfoil surface. The quasi-parallel approach assumes that at each specified location on the airfoil, the mean flow is parallel to the surface and locally invariant in the flow direction, hence only a set of ordinary differential equations need to be solved to obtain the local growth rate. Unfortunately, real boundary layers are not parallel and the surfaces of most airfoils are not flat, but contain curvature. A multiple-scales analysis (MSA) by Saric and Nayfeh [1] showed that weakly-nonparallel effects can be incorporated into the calculation of the growth rates. Masad and Malik [2] demonstrated that additional additive terms can be considered in the MSA to account for weak curvature. Incorporation of both corrections in a quasi-parallel stability program can greatly improve the results.

Here we look at the suitability of the MSA for providing instability-wave growth rates for both canonical test cases and more complicated boundary layers. In Section 2 we discuss some mathematical and computational considerations. In Section 3 we show results obtained with the MSA and compare them with results obtained from the solution of the parabolized stability equations (PSE). The significance of the results are discussed in Section 4, and finally we summarize our conclusions in Section 5.

2 Mathematical and Computational Considerations

We follow the general MSA of Saric and Nayfeh [1] for including the weakly nonparallel effects. Masad and Malik [2] show that weak surface curvature can be treated as a small perturbation to the problem without curvature. We also include this effect here. Details of the approach used are disclosed in Ref. [3]. All lengths are nondimensionalized by a length scale L^* and all

Flow	L^*	R	Sweep Angle
Swept Hiemenz	1	250	NA
Swept Circular Cylinder	0.457 m	1.18×10^6	63°
ASU Airfoil	1.294 m	1.8385×10^6	45°

Table 1: Summary of flow conditions.

velocities by a velocity scale U^* . The flow Reynolds number is $R = U^*L^*/\nu^*$ where ν^* is the dimensional kinematic viscosity. The normalized streamwise direction is denoted by x .

The numerical solution of the resultant equations involves the discretization of the equations with the use of Chebyshev polynomials. A staggered grid is used for the pressure variable and hence the continuity equation. An iterative procedure is used to determine the quasi-parallel eigenvalue and eigenfunction. Direct solves are used to solve the systems of equations. The computer code used for the calculations is a modified version of SPECLS [4].

Code validation was performed by comparison of the results with those of El-Hady [5] and Masad and Malik [2] for the nonparallel and curvature effects respectively. Singer and Choudhari [3] showed that pointwise comparison of the nonparallel results was in excellent agreement with the previously published results.

Linearized parabolized stability equations (PSE) are now commonly used to predict the linear growth of disturbances in nonparallel boundary layers. Bertolotti, Herbert, and Spalart [6] show that the linearized PSE approach reliably predicts the growth of small disturbances in a growing zero-pressure gradient boundary layer. Malik, Li, and Chang [7] arrive at the same conclusion for zero-frequency disturbances in swept Hiemenz flow by comparison with the results of linearized Navier-Stokes calculations. The PSE code used in this study was described in Malik, Li, and Chang [7]. In addition to the verification tests reported there, the code was verified against tabulated data from Bertolotti (personal communication) for the Blasius boundary layer.

3 Results

Three comparisons of MSA and PSE are considered here. In all cases we will focus on zero-frequency disturbances (i.e. stationary crossflow vortices) in three-dimensional boundary layers. The first case involves swept Hiemenz flow, i.e., flow past a swept flat plate that is placed broadside to an incident flow. This flow approximates the attachment-line region of a swept wing and was analyzed in detail by Malik, Li, and Chang [7]. For ease of presentation, the velocity scale for this flow is taken to be the constant free-stream spanwise velocity. In the other cases, the velocity scale is the total free-stream speed. The second case considered is a swept circular cylinder which matches the experimental conditions of Poll [8]. Unlike the swept Hiemenz flow, where the surface curvature is zero, the cylinder case includes both nonparallel and curvature effects. The nonparallel effects observed in this case are more severe than those in the swept Hiemenz flow. Finally, we will compare results from PSE and MSA for a mean flow computed by Streett (personal communication) to simulate the conditions of the swept-wing experiment performed at Arizona State University [9]. A summary of the parameters used in the various cases is given in Table 1.

In Fig. 1 the growth rate of the disturbance is plotted versus distance downstream of the leading edge for the swept Hiemenz flow. The solid line represents the quasi-parallel contribu-

tion. The dotted line represents the nonparallel result from MSA. The chain-dot line shows the growth rate (based on the local u_{max}) that was predicted by PSE. Note that the relatively small disagreement between the growth rates computed using MSA and PSE is almost constant over most of the flow domain.

In Fig. 2 we show the growth rates for flow over the swept circular cylinder. Both curvature and nonparallel effects are included in both the PSE and MSA. The PSE and MSA agree closely near the upstream end of the body, but the results diverge downstream. Calculations without the curvature effect included indicate that the bulk of the discrepancy between PSE and MSA is associated with the nonparallel effect.

Finally we compare results from PSE and MSA for flow over a swept wing in Fig. 3. All three results differ near the leading edge, but downstream of approximately 8% chord, the MSA and PSE results are essentially indistinguishable, with growth rates slightly less than the quasi-parallel case.

4 Discussion

The MSA is designed to be suitable for flows with weak nonparallelism. Quantitative criteria for the determination of how weak the nonparallelism must be for the MSA to perform satisfactorily have never been determined and for the general flow case, possibly never will be determined. However, we can make some progress by seeking a parameter that varies in such a way that the error trends in the MSA are correctly characterized. The nondimensional quantity

$$\Upsilon \equiv \lambda \frac{1}{l^*} \frac{dl^*}{dx}$$

where $l^* = \sqrt{\nu x / U_\infty}$ is a mean-flow length scale and λ is the streamwise wavelength of the disturbance has many of the desired properties. The parameter is the ratio of the disturbance wavelength to a length scale over which changes to the mean flow occur. When Υ is small, the mean flow does not vary significantly over the wavelength of the disturbance; hence, one would suspect that when the value of Υ is small, MSA and PSE would agree quite well. Is this suggestion supported by the data?

In the case of swept Hiemenz flow, $\frac{dl^*}{dx}$ is identically zero, even though the flow is not perfectly parallel. However, the nonparallelism is weak and, as Fig. 1 shows, the difference in MSA compared with PSE is quite small, especially when compared to the error in the quasi-parallel result. In addition, these errors remain essentially constant over the range tested.

For flow over Poll's swept circular cylinder, Υ is small near the leading edge of the cylinder; the parameter's value increases with distance along the cylinder. The errors between MSA and PSE in Fig. 2 reflect the same trends; i.e., the differences are small near the leading edge but increase further downstream. We sought to determine whether the differences between the MSA and PSE results would collapse on a single curve when plotted versus the parameter Υ . Fig. 4 shows the differences between the growth rates of the PSE and MSA normalized with the local streamwise wavelength for three different spanwise wavenumbers. Although three distinct curves are illustrated, these curves cluster together tightly and suggest satisfactory agreement of PSE and MSA in the region where $\Upsilon < 0.007$.

A similar analysis was performed for ASU airfoil data. On the ASU airfoil at -4° angle of attack, the parameter Υ is large near the leading edge, lessening in the downstream direction.

This behavior is consistent with the observed behavior of the differences between PSE and MSA in Fig. 3. The normalized differences between the growth rates from PSE and MSA for three widely spaced spanwise wavenumbers are plotted in Fig. 4. Satisfactory agreement of the PSE and MSA growth rates is observed for $\Upsilon < 0.03$. The discrepancies rapidly become large for all spanwise wave numbers when Υ exceeds 0.03. Although the value of the parameter Υ required for satisfactory results is quite different for the two different flows, within each flow, the range of values of Υ for which MSA is appropriate is not strongly influenced by the spanwise wave number.

5 Conclusions

We used MSA to approximate the effects of nonparallelism and streamwise curvature on the stability of disturbances in a variety of incompressible flows. The results of MSA were compared with results from PSE for a variety of different flows. The results suggest that the nondimensional number $\Upsilon \equiv \frac{\lambda}{l^*} \frac{dl^*}{dx}$, which is the ratio of the disturbance wavelength to the rate of change of the mean flow, provides useful information about the adequacy of the multiple-scales approximation for different disturbances for a given flow geometry. Unfortunately, the flows that we tested gave different restrictions for values of Υ required for the MSA to give satisfactory results. Because no uniform value of the number collapsed the data for the different flow geometries, the use of MSA for predicting the growth of small disturbances on new geometries is risky. However, if the growth rate history of a single disturbance computed using MSA can be checked by some other means (like PSE), then the error in the MSA result as a function of the parameter Υ can be used to estimate the range of validity of the MSA for other disturbances in the same flow.

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References

- [1] Saric, W. S. and Nayfeh, A. H., “Non-parallel stability of boundary layer flows,” *Phys. Fluids* **18**, 945 (1975).
- [2] Masad, J. A. and Malik, M. R., “Effects of body curvature and nonparallelism on the stability of flow over a swept cylinder,” *Phys. Fluids* **6**, 2363 (1994).
- [3] Singer, B. A. and Choudhari, M., “Multiple scales approach to weakly nonparallel and curvature effects: Details for the novice,” *NASA CR-198199* (1995).
- [4] Macaraeg, M. G., Streett, C. L., and Hussaini, M. Y., “A spectral collocation solution to the compressible stability eigenvalue problem,” NASA-TP 2858, December, (1988).
- [5] El-Hady, N. M., “Nonparallel instability of supersonic and hypersonic boundary layers,” *Phys. Fluids A* **3**, 2164, (1991).
- [6] Bertolotti, F. P., Herbert, Th., and Spalart, P. R., “Linear and Nonlinear Stability of the Blasius Boundary Layer,” *J. Fluid Mech.* **242**, 441, (1992).
- [7] Malik, M. R., Li, F., and C.-L. Chang, “Crossflow disturbances in three-dimensional boundary layers: Nonlinear development, wave interaction and secondary instability,” *J. Fluid Mech.* **268**, 1 (1994).
- [8] Poll, D. I. A., “Some observations of the transition process on the windward face of a long yawed cylinder,” *J. Fluid Mech.* **150**, 329 (1985).
- [9] Dagenhart, J. R., Saric, W. S., Mousseux, M. C. and Stack, J. P., “Crossflow vortex instability and transition on a 45-degree swept wing,” *AIAA Paper No. 89-1892*, (1989).

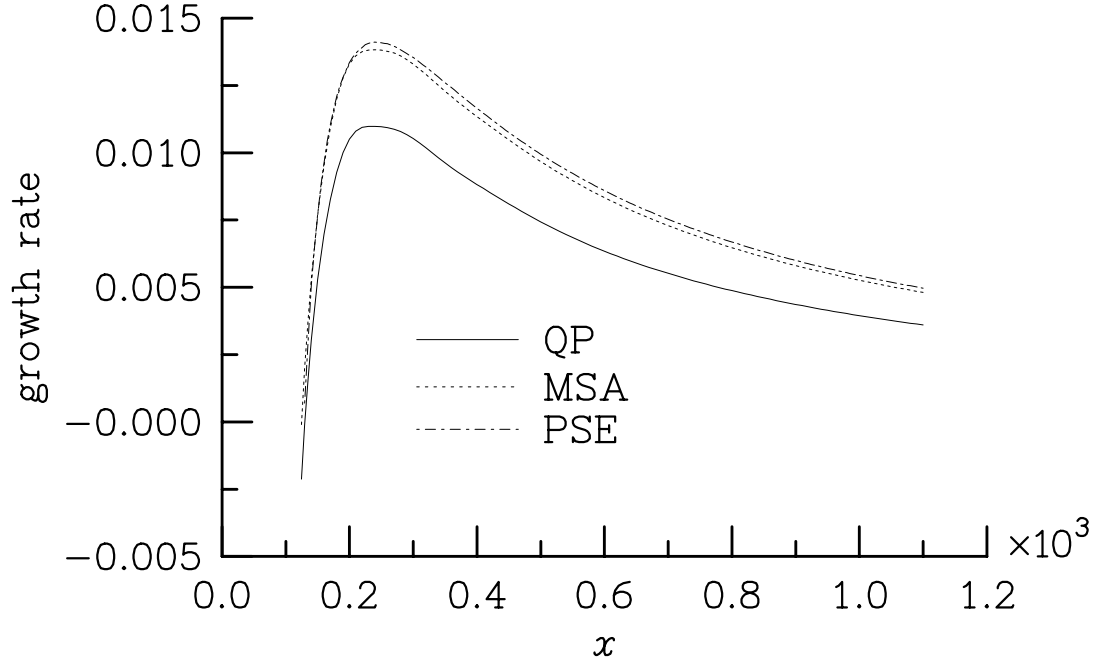


Figure 1. Growth rates for swept Hiemenz flow. $\overline{R} = 250$, $F = 0$, $\beta = 0.4$.

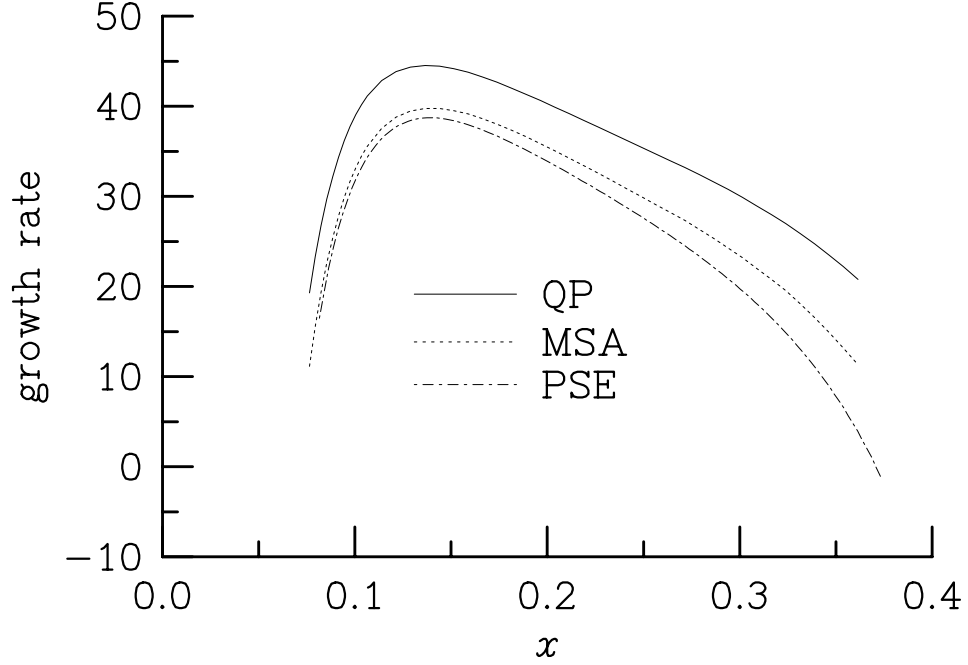


Figure 2. Growth rates for flow over Poll's swept cylinder. $R = 1.18 \times 10^6$, $F = 0$, $\beta = 0.261$.

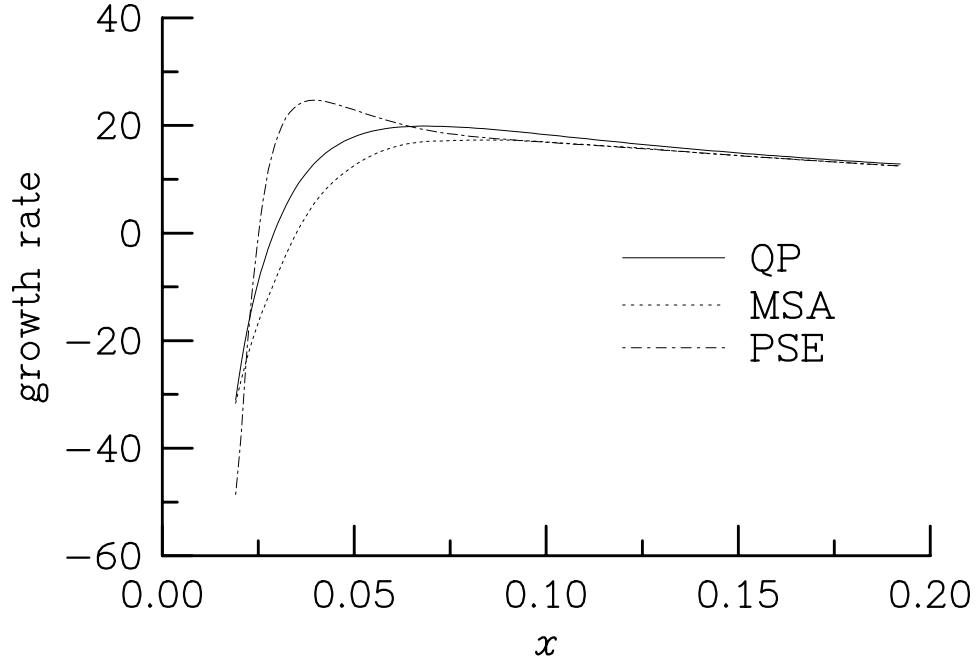


Figure 3. Growth rates along ASU airfoil at -4° angle of attack. Spanwise wave number $\beta = 900$.

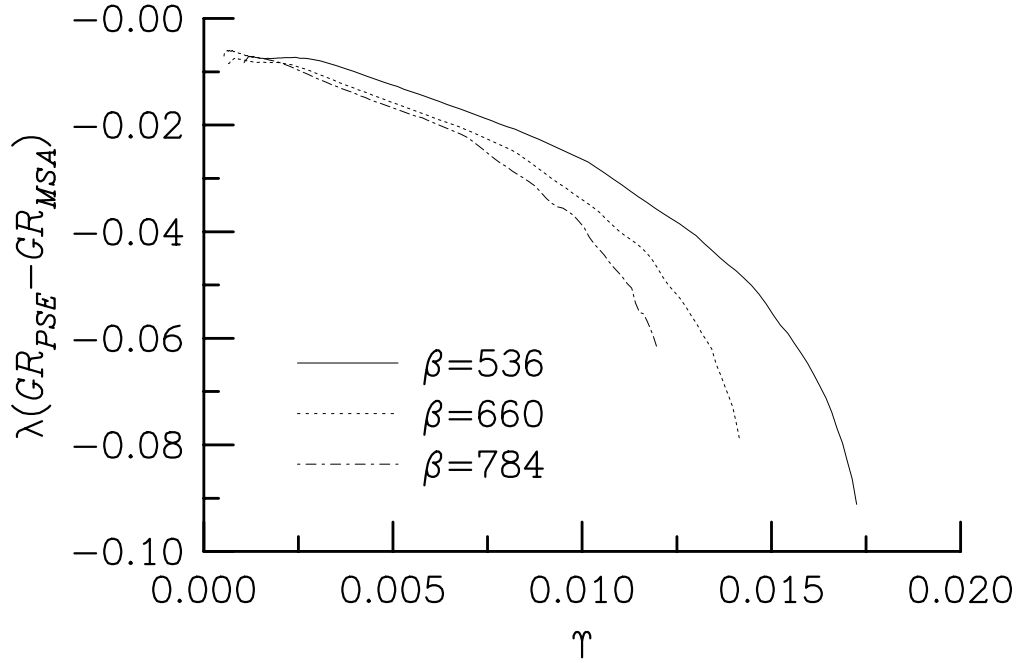


Figure 4. Normalized error in MSA versus $\Upsilon \equiv \frac{\lambda}{l^*} \frac{dl^*}{dx}$ for Poll's swept cylinder.

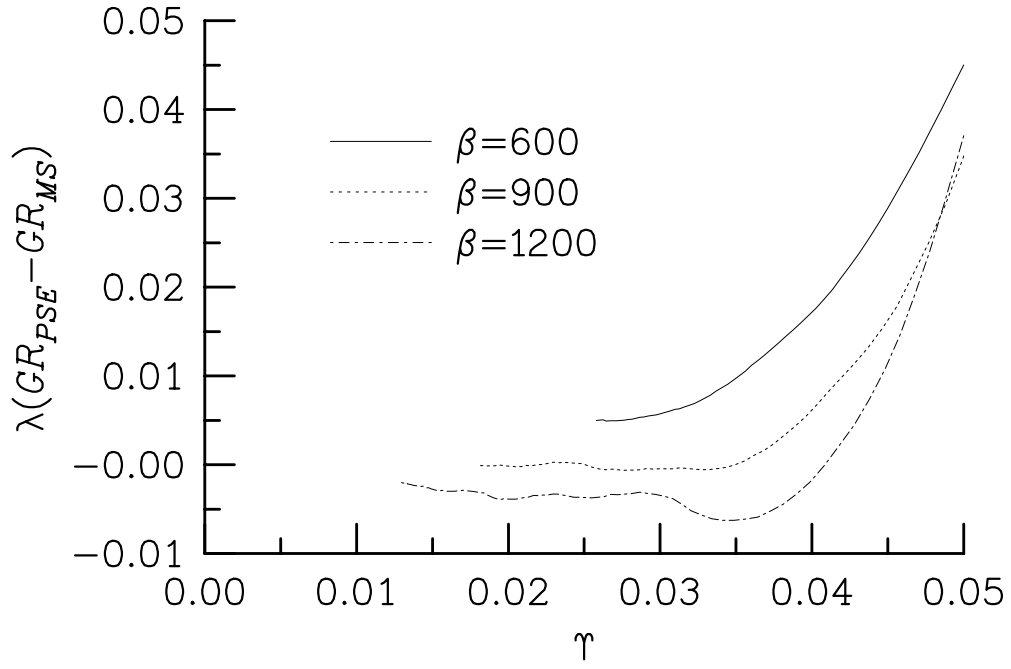


Figure 5. Normalized error in MSA versus $\Upsilon \equiv \frac{\lambda}{l^*} \frac{dl^*}{dx}$ for ASU airfoil.